

# Problems in thermodynamics and molecular physics

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1. *The density of air under normal conditions, that is, pressure  $p_0 = 101\,325\text{ Pa}$  and temperature  $t_0 = 0^\circ\text{C}$  is  $\rho_0 = 1,293\text{ kg m}^{-3}$ . What will be the density of air at pressure  $p = 0,5\text{ MPa}$  and temperature  $t = 50^\circ\text{C}$ ?*
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The problem can be solved using the equation of state of ideal gas

$$pV = nRT ,$$

where  $R$  is the universal gas constant and the amount of substance is

$$n = \frac{m}{M} ,$$

where  $M$  denotes the molar mass. The equation of state takes the form

$$pV = \frac{m}{M}RT .$$

From the density of air

$$\rho = \frac{m}{V} ,$$

it is possible to express the mass of air

$$m = \rho V ,$$

and the equation of state will take the form

$$pV = \frac{\rho V}{M}RT ,$$

from which the density can be expressed as

$$\rho = \frac{pM}{RT} .$$

The density of the gas under normal conditions will be

$$\rho_0 = \frac{p_0 M}{RT_0} .$$

By dividing the equations, the density of the gas can be expressed using the relation

$$\rho = \rho_0 \frac{pT_0}{p_0T} .$$

After inserting the numerical values

$$\rho = 1,293 \text{ kg m}^3 \cdot \frac{0,5 \cdot 10^6 \text{ Pa} \cdot 273,15 \text{ K}}{1,013 25 \cdot 10^5 \text{ Pa} \cdot 323,15 \text{ K}} = 5,39 \text{ kg m}^{-3} ,$$

where the gas temperatures in the thermodynamic scale have been converted from the Celsius scale using the relationship

$$T[\text{K}] = t[^\circ\text{C}] + 273,15 \text{ K} .$$

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2. *The pressure in the cylinder of a steam engine with volume  $V = 20 \text{ l}$  is reduced by  $\Delta p = 0,5 \text{ MPa}$  when the valve is opened. What mass of steam has been released from the cylinder if the steam temperature  $t = 100^\circ\text{C}$  has not changed?*
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The solution of the problem is possible using the equation of state of ideal gas

$$pV = nRT ,$$

where  $R$  is the universal gas constant and the amount of substance is

$$n = \frac{m}{M} ,$$

where  $M$  denotes the molar mass. The equation of state takes the form

$$pV = \frac{m}{M}RT .$$

Before the steam was released, the equation of state described the gas is

$$p_1V = \frac{m_1}{M}RT ,$$

After the steam was released, the equation of state described the gas is

$$p_2V = \frac{m_2}{M}RT .$$

By subtracting the equations of state it is possible to obtain the equation

$$(p_1 - p_2)V = (m_1 - m_2)\frac{RT}{M} ,$$

in which the mass of the released steam is

$$\Delta m = m_1 - m_2 ,$$

and the reduction of pressure is

$$\Delta p = p_1 - p_2 .$$

The equation can therefore be written as

$$\Delta p V = \Delta m \frac{RT}{M} ,$$

from which it is possible to express the mass of the released steam

$$\Delta m = \frac{MV}{RT} \Delta p ,$$

and after inserting the numerical values

$$\Delta m = \frac{0,018 \text{ kg mol}^{-1} \cdot 0,02 \text{ m}^3}{8,314 \text{ J K}^{-1} \text{ mol}^{-1} \cdot 373,15 \text{ K}} \cdot 5 \cdot 10^5 \text{ Pa} = 0,058 \text{ kg} = 58 \text{ g} ,$$

where the gas temperatures in the thermodynamic scale have been converted from the Celsius scale using the relationship

$$T[\text{K}] = t[^\circ\text{C}] + 273,15 \text{ K} .$$

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3. *The same gas is in two containers separated by a cap. In the first container with volume  $V_1 = 2\text{ l}$  the pressure of the gas is  $p_1 = 0,2 \text{ MPa}$ . In the second container with volume  $V_2 = 4\text{ l}$  the pressure of the gas is  $p_2 = 0,4 \text{ MPa}$ . What will be the resulting pressure, when the cap is opened, if the temperature in containers was the same and it will remain the same after joining?*
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From the equation of state for the gas in the first container

$$p_1 V_1 = n_1 RT ,$$

follows the amount of of gas in the first container is

$$n_1 = \frac{p_1 V_1}{RT} .$$

From the equation of state for the gas in the second container

$$p_2 V_2 = n_2 RT ,$$

follows the amount of gas in the second container is

$$n_2 = \frac{p_2 V_2}{RT} .$$

When the cap is opened, the equation of state for the gas will be

$$p(V_1 + V_2) = (n_1 + n_2)RT ,$$

when substituted for the amounts of substance, the equation of state takes the form

$$p(V_1 + V_2) = \left( \frac{p_1 V_1}{RT} + \frac{p_2 V_2}{RT} \right) RT ,$$

which can be simplified to the equation

$$p(V_1 + V_2) = p_1 V_1 + p_2 V_2 ,$$

from which the resulting pressure can be calculated as

$$p = \frac{p_1 V_1 + p_2 V_2}{V_1 + V_2} ,$$

and after inserting the numerical values, the resulting pressure is

$$p = \frac{0,2 \cdot 10^6 \text{ Pa} \cdot 2 \cdot 10^{-3} \text{ m}^3 + 0,4 \cdot 10^6 \text{ Pa} \cdot 4 \cdot 10^{-3} \text{ m}^3}{2 \cdot 10^{-3} \text{ m}^3 + 4 \cdot 10^{-3} \text{ m}^3} = 0,33 \cdot 10^6 \text{ Pa} .$$

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4. *What is the internal energy of nitrogen, which at pressure  $p = 0,5 \text{ MPa}$  occupies volume  $V = 5 \text{ l}$ ?*
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The internal energy of a gas is the sum of the kinetic energies of all  $N$  gas molecules

$$U = N \epsilon_s ,$$

where the equipartition theorem for the mean kinetic energy of one molecule implies

$$\epsilon_s = \frac{i}{2} kT ,$$

where  $i$  denotes the number of degrees of freedom of the molecule and  $k$  is the Boltzmann constant. Thus, the kinetic energy of all molecules will be

$$U = \frac{i}{2} N kT .$$

The number of gas molecules will be

$$N = n N_A ,$$

where  $N_A$  is Avogadro's constant and the expression for Boltzmann's constant is

$$k = \frac{R}{N_A},$$

therefore the internal energy of a gas can also be expressed as

$$U = \frac{i}{2}nRT.$$

Using the equation of state

$$pV = nRT,$$

the internal energy of a gas can be expressed in terms of pressure and volume as

$$U = \frac{i}{2}pV,$$

Since nitrogen is a diatomic gas,  $i = 5$  and the internal energy of the gas will be

$$U = \frac{5}{2}pV,$$

and after inserting the numerical values, the internal energy is

$$U = \frac{5}{2} \cdot 0,5 \cdot 10^6 \text{ Pa} \cdot 5 \cdot 10^{-3} \text{ m}^3 = 6250 \text{ J}.$$

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5. *How does the mean kinetic energy of an argon gas molecule with mass  $m = 500 \text{ g}$  change if we supply the gas with heat  $Q = 5000 \text{ J}$  and at the same time the gas does work  $A' = 2000 \text{ J}$ ? The molar mass of argon is  $M = 39,9 \text{ g mol}^{-1}$ .*
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The internal energy of a gas is the sum of the kinetic energies of all  $N$  gas molecules

$$U = N\epsilon_s,$$

therefore the change in the internal energy of the gas will also be the change in the mean kinetic energy of all the molecules

$$\Delta U = N\Delta\epsilon_s.$$

The number of molecules can be calculated from the amount of substance

$$N = nN_A,$$

which can be calculated from the mass of the gas

$$n = \frac{m}{M}.$$

The change in the internal energy of a gas can therefore be written as

$$\Delta U = \frac{mN_A}{M} \Delta \epsilon_s,$$

from which the change in the mean kinetic energy of the molecule will be

$$\Delta \epsilon_s = \frac{M}{mN_A} \Delta U.$$

According to the first law of thermodynamics

$$\Delta U = Q + A,$$

the change in the internal energy of the gas is equal to the sum of the heat input and the external mechanical work. If the work is done by the gas

$$A = -A',$$

the change in internal energy will be

$$\Delta U = Q - A'.$$

The change in the mean kinetic energy of a molecule can thus be expressed as

$$\Delta \epsilon_s = \frac{M}{mN_A} (Q - A')$$

and after inserting the numerical values

$$\Delta \epsilon_s = \frac{39,9 \cdot 10^{-3} \text{ kg mol}^{-1}}{0,5 \text{ kg} \cdot 6,022 \cdot 10^{23} \text{ mol}^{-1}} \cdot (5000 \text{ J} - 2000 \text{ J}) = 3,98 \cdot 10^{-22} \text{ J}.$$

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6. A container of volume  $V = 0,05 \text{ m}^3$  contains hydrogen at temperature  $t_0 = 27^\circ\text{C}$  and pressure  $p_0 = 100 \text{ kPa}$ . Calculate the pressure and temperature of the gas if the hydrogen received heat  $Q = 1,5 \text{ kJ}$ .
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The change in the internal energy of a gas can be expressed using the temperature difference as

$$\Delta U = \frac{i}{2} Nk \Delta T = \frac{i}{2} Nk (T_2 - T_1),$$

where the number of particles can be calculated using the amount of substance

$$N = nN_A$$

and the Boltzmann constant is

$$k = \frac{R}{N_A},$$

this implies for the change of the internal energy of the gas

$$\Delta U = \frac{i}{2}nR(T_2 - T_1).$$

Using the equation of state

$$p_0V = nRT_0,$$

$$pV = nRT,$$

the change of the internal energy can be written in the form

$$\Delta U = \frac{i}{2}(pV - p_0V).$$

According to the first law of thermodynamics

$$\Delta U = Q + A,$$

the change in the internal energy of the gas is equal to the sum of the heat input and the external mechanical work. If the volume of the gas is constant, the mechanical work is zero

$$dA = -pdV \implies A = 0$$

and the change in internal energy is equal to the heat input

$$\Delta U = Q.$$

It is therefore valid

$$Q = \frac{i}{2}(pV - p_0V),$$

from which the gas pressure can be calculated

$$p = \frac{Q + \frac{i}{2}p_0V}{\frac{i}{2}V}.$$

Because hydrogen is a diatomic gas, the number of degrees of freedom of its molecule is  $i = 5$  and

$$p = \frac{1500 \text{ J} + \frac{5}{2} 100\,000 \text{ Pa} \cdot 0,05 \text{ m}^3}{\frac{5}{2} \cdot 0,05 \text{ m}^3} = 112\,000 \text{ Pa} = 112 \text{ kPa} .$$

For the isochoric process, the Charles' law states

$$\frac{p_0}{T_0} = \frac{p}{T} ,$$

which gives the resulting temperature

$$T = \frac{p}{p_0} T_0$$

and after inserting the numerical values

$$T = \frac{112\,000 \text{ Pa}}{100\,000 \text{ Pa}} \cdot 300,15 \text{ K} = 336,17 \text{ K} = 63,02^\circ\text{C} .$$

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7. In nitrogen with mass  $m = 200 \text{ g}$ , initial temperature  $t_1 = 27^\circ\text{C}$  and pressure  $p_1 = 0,4 \text{ MPa}$ , a thermodynamic process took place in which the pressure of nitrogen decreased to  $p_2 = 0,3 \text{ MPa}$ . How much heat was added to the nitrogen, what work did the gas do, and how did its internal energy change if the process was a) isochoric, b) isothermal, c) adiabatic? Draw all these processes in  $p$ - $V$  diagrams.

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a) For isochoric process, the volume of the gas is constant

$$V = \text{konšt.} \implies dV = 0 ,$$

which implies that the work of the gas is zero

$$dA' = p dV \implies A' = 0 .$$

According to the first law of thermodynamics, the heat delivered to a gas is equal to the sum of the change in its internal energy and the work done by the gas

$$Q = \Delta U + A' ,$$

thus, for isochoric process

$$Q = \Delta U .$$



The change in the internal energy of a gas can be expressed in terms of the temperature change as

$$\Delta U = \frac{i}{2} n R \Delta T = \frac{i}{2} \frac{m}{M} R (T_2 - T_1),$$

where  $M = 28 \text{ g mol}^{-1}$  is the molar mass of nitrogen. From the Charles' law

$$\frac{p_1}{T_1} = \frac{p_2}{T_2},$$

follows

$$T_2 = \frac{p_2}{p_1} T_1$$

and the change in internal energy can be expressed as

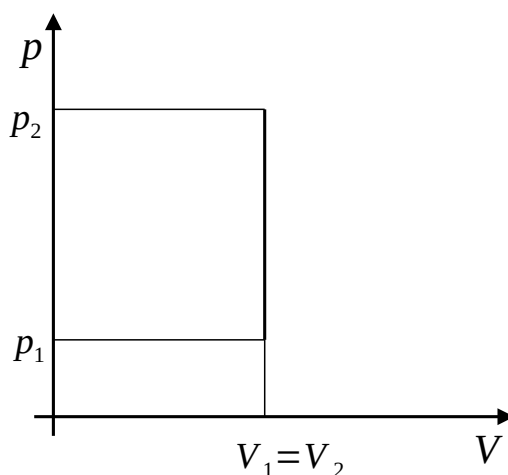
$$\Delta U = \frac{i}{2} \frac{m}{M} R T_1 \left( \frac{p_2}{p_1} - 1 \right),$$

after inserting the numerical values

$$\begin{aligned} \Delta U &= \frac{5 \cdot 0,2 \text{ kg}}{2 \cdot 28 \cdot 10^{-3} \text{ kg mol}^{-1}} \cdot 8,314 \text{ J K}^{-1} \text{ mol}^{-1} \cdot 300,15 \text{ K} \left( \frac{0,3 \cdot 10^6 \text{ Pa}}{0,4 \cdot 10^6 \text{ Pa}} - 1 \right) = \\ &= -11\,140 \text{ J} = -11,14 \text{ kJ}. \end{aligned}$$

The heat added to the gas is equal to the change in internal energy

$$Q = \Delta U = -11,14 \text{ kJ}.$$



Obr. 1

b) For isothermal process, the temperature of the gas is constant

$$T = \text{konšt.} \implies \Delta T = 0 ,$$

therefore the change in internal energy of the gas is zero

$$\Delta U = \frac{i}{2} n R \Delta T = 0 ,$$

and from the first law of thermodynamics

$$Q = \Delta U + A' ,$$

follows

$$Q = A' .$$

From the Boyle-Mariott law

$$pV = p_1 V_1 ,$$

follows

$$p = \frac{p_1 V_1}{V} ,$$

and from the equation of state

$$p_1 V_1 = n R T_1 ,$$

follows

$$p = \frac{n R T_1}{V} = \frac{m}{M} \frac{R T_1}{V} .$$

The work of the gas can be calculated as

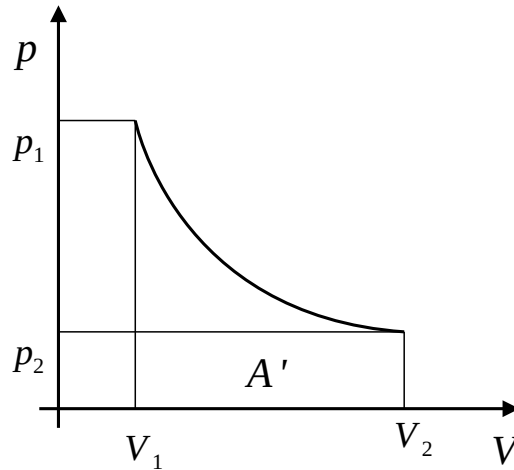
$$\begin{aligned} A' &= \int_{V_1}^{V_2} p dV = \frac{m}{M} R T_1 \int_{V_1}^{V_2} \frac{1}{V} dV = \frac{m}{M} R T_1 [\ln V]_{V_1}^{V_2} = \\ &= \frac{m}{M} R T_1 (\ln V_2 - \ln V_1) = \frac{m}{M} R T_1 \ln \frac{V_2}{V_1} = \frac{m}{M} R T_1 \ln \frac{p_1}{p_2} . \end{aligned}$$

After inserting the numerical values

$$\begin{aligned} A' &= \frac{0,2 \text{ kg}}{28 \cdot 10^{-3} \text{ g mol}^{-1}} \cdot 8,314 \text{ J K mol}^{-1} \cdot 300,15 \text{ K} \cdot \ln \frac{0,4 \cdot 10^6 \text{ Pa}}{0,3 \cdot 10^6 \text{ Pa}} = \\ &= 5128 \text{ J} = 5,128 \text{ kJ} . \end{aligned}$$

The heat added to the gas is equal to the work of the gas

$$Q = A' = 5,128 \text{ kJ} .$$



Obr. 2

c) In adiabatic process, there is no heat exchange between the gas and the surroundings

$$Q = 0 .$$

From the first law of thermodynamics

$$Q = \Delta U + A' ,$$

follows

$$\Delta U = -A' .$$

The change in the internal energy of a gas can be expressed as

$$\Delta U = \frac{i}{2} \frac{m}{M} R (T_2 - T_1) = \frac{i}{2} \frac{m}{M} R T_1 \left( \frac{T_2}{T_1} - 1 \right) .$$

From the Poisson's equation

$$p_1 V_1^\kappa = p_2 V_2^\kappa ,$$

can be expressed

$$\frac{p_1}{p_2} = \left( \frac{V_2}{V_1} \right)^\kappa \implies \left( \frac{p_1}{p_2} \right)^{\frac{1}{\kappa}} = \frac{V_2}{V_1} ,$$

and from the equation of state

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} ,$$

can be expressed

$$\frac{T_2}{T_1} = \frac{p_2 V_2}{p_1 V_1},$$

which can be transformed in to the form

$$\frac{T_2}{T_1} = \left(\frac{p_1}{p_2}\right)^{-1} \left(\frac{p_1}{p_2}\right)^{\frac{1}{\kappa}} = \left(\frac{p_1}{p_2}\right)^{\frac{1-\kappa}{\kappa}}.$$

Using the Mayer's equation

$$C_p = C_v + R,$$

the Poisson constant can be expressed in terms of the number of degrees of freedom

$$\kappa = \frac{C_p}{C_v} = \frac{C_v + R}{C_v} = \frac{\frac{i}{2}R + R}{\frac{i}{2}R} = \frac{i+2}{i}.$$

The change of internal energy can therefore be calculated as

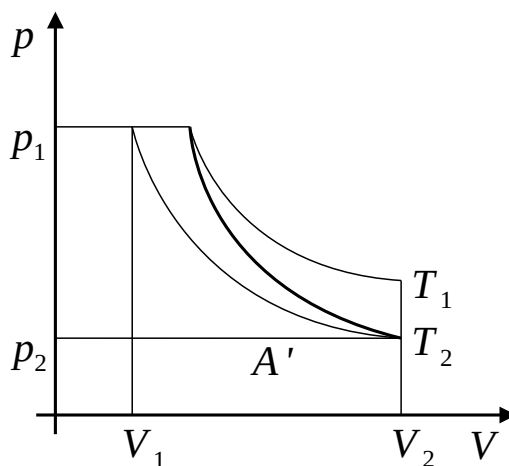
$$\Delta U = \frac{i}{2} \frac{m}{M} R T_1 \left[ \left(\frac{p_1}{p_2}\right)^{-\frac{2}{i+2}} - 1 \right] = \frac{i}{2} \frac{m}{M} R T_1 \left[ \left(\frac{p_2}{p_1}\right)^{\frac{2}{i+2}} - 1 \right].$$

Inserting numerical values

$$\begin{aligned} \Delta U &= \frac{5 \cdot 0,2 \text{ kg}}{2 \cdot 28 \cdot 10^{-3} \text{ kg mol}^{-1}} \cdot 8,314 \text{ J K}^{-1} \cdot 300,15 \text{ K} \left[ \left(\frac{0,3 \cdot 10^6 \text{ Pa}}{0,4 \cdot 10^6 \text{ Pa}}\right)^{\frac{2}{7}} - 1 \right] = \\ &= -3516 \text{ J} = -3,516 \text{ kJ}. \end{aligned}$$

The work of the gas will be

$$A' = -\Delta U = 3,516 \text{ kJ}.$$



Obr. 3

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8. To air of temperature  $t_1 = 20^\circ\text{C}$ , which occupies at pressure  $p_1 = 0,1\text{ MPa}$  a volume  $V_1 = 2\text{ m}^3$ , a heat  $Q = 400\text{ kJ}$  has been added. Calculate the change in internal energy, external work and final state quantities if the action was a) isochoric b) isobaric c) isothermal. The Poisson constant of air is  $\kappa = 1,4$ .
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a) In isochoric process the volume of a gas is constant, therefore the gas does not produce the mechanical work

$$V = \text{konšt.} \implies A' = 0$$

and from the first law of thermodynamics

$$Q = \Delta U + A' ,$$

follows that the change in internal energy of a gas is equal to the added heat

$$\Delta U = Q = 400\text{ kJ} .$$

From the equation of state

$$p_1 V_1 = n R T_1 ,$$

the amount of substance can be expressed as

$$n = \frac{p_1 V_1}{R T_1}$$

and from Poisson's constant

$$\kappa = \frac{i + 2}{i} ,$$

the number of degrees of freedom is

$$i = \frac{2}{\kappa - 1} .$$

The change of the internal energy of a gas can be expressed as

$$\Delta U = \frac{i}{2} n R (T_2 - T_1) = \frac{1}{\kappa - 1} \frac{p_1 V_1}{T_1} (T_2 - T_1) ,$$

because

$$Q = \frac{1}{\kappa - 1} \frac{p_1 V_1}{T_1} (T_2 - T_1) ,$$

the resulting temperature can be calculated as

$$T_2 = \frac{Q + \frac{1}{\kappa-1}p_1V_1}{\frac{1}{\kappa-1} \frac{p_1V_1}{T_1}} = T_1 \left[ \frac{(\kappa-1)Q}{p_1V_1} + 1 \right]$$

and after inserting the numerical values

$$T_2 = 293,15 \text{ K} \cdot \left[ \frac{(1,4-1) \cdot 400 \cdot 10^3 \text{ J}}{0,1 \cdot 10^6 \text{ Pa} \cdot 2 \text{ m}^3} + 1 \right] = 527,67 \text{ K} .$$

From Charles's law

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} ,$$

follows the resulting pressure

$$p_2 = p_1 \frac{T_2}{T_1} ,$$

which can be modified to the form

$$p_2 = p_1 \left[ \frac{(\kappa-1)Q}{p_1V_1} + 1 \right] ,$$

and after inserting the numerical values

$$p_2 = T_2 = 0,1 \cdot 10^6 \text{ Pa} \cdot \left[ \frac{(1,4-1) \cdot 400 \cdot 10^3 \text{ J}}{0,1 \cdot 10^6 \text{ Pa} \cdot 2 \text{ m}^3} + 1 \right] = 0,18 \cdot 10^6 \text{ Pa} .$$

b) In isobaric process the pressure of the gas is constant

$$p = \text{konšt.} .$$

The amount of substance is

$$n = \frac{p_1V_1}{RT_1}$$

and the number of degrees of freedom is

$$i = \frac{2}{\kappa-1} .$$

The change of internal energy of gas can then be expressed as

$$\Delta U = \frac{i}{2}nR(T_2 - T_1) = \frac{1}{\kappa-1} \frac{p_1V_1}{RT_1} R(T_2 - T_1) = \frac{1}{\kappa-1} p_1V_1 \left( \frac{T_2}{T_1} - 1 \right) .$$

From the Gay-Lussac's law

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} ,$$

follows

$$\frac{T_2}{T_1} = \frac{V_2}{V_1},$$

by which the change of internal energy can be calculated as

$$\Delta U = \frac{1}{\kappa - 1} p_1 V_1 \left( \frac{V_2}{V_1} - 1 \right).$$

The work of the gas at constant pressure is

$$A' = \int_{V_1}^{V_2} p_1 dV = p_1 (V_2 - V_1).$$

From the first law of thermodynamics

$$Q = \Delta U + A',$$

follows

$$Q = \frac{1}{\kappa - 1} p_1 V_1 \left( \frac{V_2}{V_1} - 1 \right) + p_1 (V_2 - V_1),$$

which can be modified to the form

$$Q(\kappa - 1) + p_1 V_1 \kappa = p_1 V_2 \kappa,$$

from which the resulting volume can be expressed as

$$V_2 = V_1 \left[ \frac{Q(\kappa - 1)}{\kappa p_1 V_1} + 1 \right],$$

after inserting the numerical values

$$V_2 = 2 \text{ m}^3 \cdot \left[ \frac{400 \cdot 10^3 \text{ J} \cdot (1,4 - 1)}{1,4 \cdot 0,1 \cdot 10^6 \text{ Pa} \cdot 2 \text{ m}^3} + 1 \right] = 3,14 \text{ m}^3. \text{ From the Gay - Lussac's law}$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2},$$

for the resulting temperature follows

$$T_2 = T_1 \frac{V_2}{V_1},$$

which can be transformed to the form

$$T_2 = T_1 \left[ \frac{Q(\kappa - 1)}{\kappa p_1 V_1} + 1 \right]$$

and after inserting the numerical values

$$T_2 = 293,15 \text{ K} \cdot \left[ \frac{400 \cdot 10^3 \text{ J} \cdot (1,4 - 1)}{1,4 \cdot 0,1 \cdot 10^6 \text{ Pa} \cdot 2 \text{ m}^3} + 1 \right] = 460,66 \text{ K} .$$

c) In isothermal process the temperature of the gas is constant

$$T = \text{konšt.} ,$$

therefore the change of the internal energy of the gas is zero

$$\Delta U = 0 .$$

From the first law of thermodynamics

$$Q = \Delta U + A' ,$$

follows that the work of the gas is equal to the added heat

$$A' = Q = 400 \text{ kJ} .$$

From the Boyle-Mariott's law

$$p_1 V_1 = p_2 V_2 = pV ,$$

it is possible to express the pressure of a gas

$$p = \frac{p_1 V_1}{V} ,$$

which can be used to express the work of the gas

$$A' = \int_{V_1}^{V_2} p dV = \int_{V_1}^{V_2} \frac{p_1 V_1}{V} dV = [p_1 V_1 \ln V]_{V_1}^{V_2} = p_1 V_1 \ln \frac{V_2}{V_1} .$$

Because it applies

$$p_1 V_1 \ln \frac{V_2}{V_1} = Q ,$$

the resulting volume will be

$$V_2 = V_1 e^{\frac{Q}{p_1 V_1}} ,$$

after inserting the numerical values

$$V_2 = 2 \text{ m}^3 \cdot e^{\frac{400 \cdot 10^3 \text{ J}}{0,1 \cdot 10^6 \text{ Pa} \cdot 2 \text{ m}^3}} = 14,78 \text{ m}^3 .$$



The resulting pressure will then be

$$p_2 = p_1 \frac{V_1}{V_2} = p_1 e^{-\frac{Q}{p_1 V_1}}$$

and after adding the numerical values

$$p_2 = 0,1 \cdot 10^6 \text{ Pa} \cdot e^{\frac{-400 \cdot 10^3 \text{ J}}{0,1 \cdot 10^6 \text{ Pa} \cdot 2 \text{ m}^3}} = 13\,533 \text{ Pa}.$$

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9. Helium with the amount of substance  $n = 2$  kmol expands isobarically and increases its volume threefold. What is the change of entropy for this action?
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The change of entropy for reversible processes is defined as

$$dS = \frac{dQ}{T},$$

for an isobaric event, when the system goes from the 1 state to the 2 state, the total entropy change will be

$$\Delta S = \int_1^2 \frac{dQ}{T} = \int_1^2 \frac{nC_p dT}{T}.$$

The molar heat capacity at constant volume is

$$C_V = \frac{i}{2}R,$$

using Mayer's relation

$$C_p = C_V + R,$$

the molar heat capacity at constant pressure can be expressed as

$$C_p = \frac{i}{2}R + R = \frac{i+2}{2}R.$$

The entropy change will then be

$$\Delta S = n \frac{i+2}{2} R \int_1^2 \frac{dT}{T} = n \frac{i+2}{2} R [\ln T]_{T_1}^{T_2} = n \frac{i+2}{2} R \ln \frac{T_2}{T_1}.$$

From the Gay-Lussac's law

$$\frac{V_1}{T_1} = \frac{V_2}{T_2},$$

follows

$$\frac{T_2}{T_1} = \frac{V_2}{V_1},$$

by which the entropy change can be expressed as

$$\Delta S = n \frac{i+2}{2} R \ln \frac{V_2}{V_1}$$

and after adding the numerical values

$$\Delta S = 2 \cdot 10^3 \text{ mol} \cdot \frac{3+2}{2} \cdot 8,314 \text{ J K}^{-1} \text{ mol}^{-1} \cdot \ln 3 = 45\,669 \text{ J K}^{-1} .$$

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10. Calculate the change of entropy of an ideal gas that is isothermally expanded from a volume  $V_0 = 2\text{ l}$  into a vacuum to a total volume  $V_1 = 8\text{ l}$ . The gas is helium with mass  $m = 20\text{ g}$ .
- 

The change of entropy is defined as

$$dS = \frac{dQ}{T} .$$

From the first law of thermodynamics

$$dQ = dU + dA' = nC_v dT + p dV ,$$

for the isothermal process

$$dT = 0 ,$$

follows

$$dQ = p dV .$$

From the equation of state

$$pV = nRT$$

it is possible to express the pressure of the gas

$$p = \frac{nRT}{V} ,$$

by which the first law of thermodynamics will have the form

$$dQ = \frac{nRT}{V} dV$$

and the change of entropy can be expressed as

$$dS = \frac{nRT}{V} \frac{dV}{T} = nR \frac{dV}{V} .$$

The amount of substance will be

$$n = \frac{m}{M},$$

thus the entropy change can be written as

$$dS = \frac{m}{M} R \frac{dV}{V}$$

and the total change of entropy in isothermal expansion will be

$$\Delta S = \frac{m}{M} R \int_{V_0}^{V_1} \frac{dV}{V} = \frac{m}{M} R [\ln V]_{V_0}^{V_1} = \frac{m}{M} R \ln \frac{V_1}{V_0},$$

after inserting the numerical values

$$\Delta S = \frac{0,02 \text{ kg}}{4 \cdot 10^{-3} \text{ kg mol}^{-1}} \cdot 8,314 \text{ J K}^{-1} \text{ mol}^{-1} \cdot \ln \frac{8 \cdot 10^{-3} \text{ m}^3}{2 \cdot 10^{-3} \text{ m}^3} = 57,63 \text{ J K}^{-1}.$$