Problems in electric field and electric current

1. *In a vacuum, there are two balls at a distance* $d = 10\,\mathrm{cm}$*, from each other, which have electric charges* $Q_1 = 20 \cdot 10^{-6}\,\mathrm{C}$ *and* $Q_2 = -10 \cdot 10^{-6}\,\mathrm{C}$*. What force are they attracted and what force will they repel each other when do they touch and then move apart again to their original distance?*

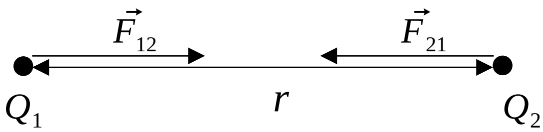

Fig. 1

The force exerted by the ball with charge Q_1 on the ball with charge Q_2 is given by Coulomb's law

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0}\frac{Q_1Q_2}{r^3}\vec{r}_{12}\ ,$$

where $\epsilon_0 = 8{,}854\cdot10^{-12}\,\mathrm{C}^2\,\mathrm{N}^{-1}\,\mathrm{m}^{-2}$ is the electric constant and $\vec{r}_{12}$ is the position vector of the ball with charge Q_2 with respect to the ball with charge Q_1. Likewise, the force exerted by the ball with charge Q_2 on the ball with charge Q_1 is given by Coulomb's law

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0}\frac{Q_1Q_2}{r^3}\vec{r}_{21}\ ,$$

where $\vec{r}_{21}$ is the position vector of the ball with charge Q_1 with respect to the ball with charge Q_2. The forces have an attractive direction and from Coulomb's law for their magnitudes follows

$$F_{12} = F_{21} = \frac{1}{4\pi\epsilon_0}\frac{|Q_1||Q_2|}{r^2}\ ,$$

after inserting numerical values

$$F_{12} = F_{21} = \frac{1}{4\pi\ .\ 8{,}854\cdot10^{-12}\,\mathrm{C}^2\,\mathrm{N}^{-1}\,\mathrm{m}^{-2}}\ .\ \frac{20\cdot10^{-6}\,\mathrm{C}\ .\ 10\cdot10^{-6}\,\mathrm{C}}{(0{,}1\,\mathrm{m})^2} = 179{,}8\,\mathrm{N}\ .$$

If the balls touch, the resulting electric charge will be

$$Q = Q_1 + Q_2 = 20\cdot10^{-6}\,\mathrm{C} + (-10\cdot10^{-6}\,\mathrm{C}) = 10\cdot10^{-6}\,\mathrm{C}$$

and after separating the balls, on each ball remains the same electric charge

$$Q^* = \frac{Q}{2} = \frac{10 \cdot 10^{-6}\,\mathrm{C}}{2} = 5 \cdot 10^{-6}\,\mathrm{C}\,.$$

The forces will have a repulsive direction and from Coulomb's law for their magnitudes follows

$$F^*_{12} = F^*_{21} = \frac{1}{4\pi\epsilon_0}\frac{|Q^*||Q^*|}{r^2}\,,$$

after inserting numerical values

$$F^*_{12} = F^*_{21} = \frac{1}{4\pi\,.\,8{,}854 \cdot 10^{-12}\,\mathrm{C}^2\,\mathrm{N}^{-1}\,\mathrm{m}^{-2}} \cdot \frac{5 \cdot 10^{-6}\,\mathrm{C}\,.\,5 \cdot 10^{-6}\,\mathrm{C}}{(0{,}1\,\mathrm{m})^2} = 22{,}5\,\mathrm{N}\,.$$

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2. *Calculate the electric potential and intensity of the electric field of a rod with electric charge Q and length l at a point lying on the extension of the rod at a distance a from its end.*

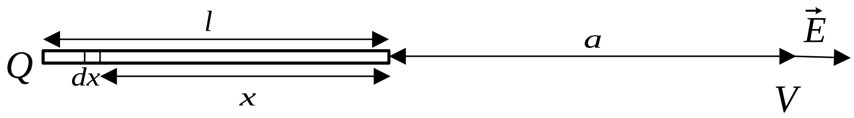

Fig. 2

The linear density of electric charge of the rod is

$$\lambda = \frac{Q}{l}\,,$$

therefore, the element of electric charge of the rod will be

$$\mathrm{d}Q = \lambda \mathrm{d}x = \frac{Q}{l}\mathrm{d}x$$

and the electric potential of this element at a point located at a distance a from the end of the rod will be

$$\mathrm{d}V = \frac{1}{4\pi\epsilon_0}\frac{\mathrm{d}Q}{x+a} = \frac{1}{4\pi\epsilon_0}\frac{Q}{l}\frac{\mathrm{d}x}{x+a}\,.$$

The electric potential of the entire rod can be calculated by integrating over the entire electric charge of the rod as

$$V = \int_Q \mathrm{d}V = \frac{1}{4\pi\epsilon_0}\frac{Q}{l}\int_0^l \frac{\mathrm{d}x}{x+a} = \frac{1}{4\pi\epsilon_0}\frac{Q}{l}\left[\ln(x+a)\right]_0^l = \frac{1}{4\pi\epsilon_0}\frac{Q}{l}\ln\frac{l+a}{a}\,.$$

The relationship between electric field and electric potential is

$$\vec{E} = -\text{grad}\, V \ ,$$

therefore, the electric field of the rod will be:

$$\vec{E} = -\frac{\mathrm{d}V}{\mathrm{d}a}\vec{\rho} = -\frac{1}{4\pi\epsilon_0}\frac{Q}{l}\frac{a}{l+a}\frac{a-l-a}{a^2}\vec{\rho} = \frac{1}{4\pi\epsilon_0}\frac{Q}{a(l+a)}\vec{\rho}\ ,$$

where $\vec{\rho}$ is a unit vector in the direction of the electric field, and the rules for the derivative of a composite function and the derivative of a fraction of functions were used.

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3. *A particle with an electric charge* $Q' = 5\,\mu\mathrm{C}$ *is located in a vacuum at a distance* $r = 2\,\mathrm{cm}$ *from an electrically charged straight thin conductor with a linear electric charge density* $\lambda = 3\,\mu\mathrm{C\,m^{-1}}$. *What is the electric force acting on this particle?*

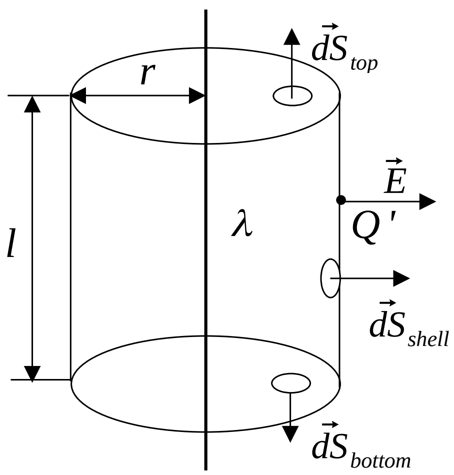

Fig. 3

According to Gauss' law of electrostatics, the electric flux through any closed surface is equal to the ratio of the electric charge inside the closed surface and the electric constant

$$\oint_S \vec{E}\cdot\mathrm{d}\vec{S} = \frac{Q}{\epsilon_0}\ .$$

If the closed surface is chosen as the surface of a cylinder whose axis of symmetry is located on the electric conductor and the radius of the cylinder is equal to the distance of the particle from the electric conductor, the total electric flux can be expressed as the sum of the electric fluxes through the upper base, the lower base and the shell of the cylinder

$$\oint_S \vec{E}\cdot\mathrm{d}\vec{S} = \int_{S_\text{top}} \vec{E}\cdot\mathrm{d}\vec{S}_\text{top} + \int_{S_\text{bottom}} \vec{E}\cdot\mathrm{d}\vec{S}_\text{bottom} + \int_{S_\text{shell}} \vec{E}\cdot\mathrm{d}\vec{S}_\text{shell}\ ,$$

because both $\mathrm{d}\vec{S}_{\text{top}}$ and $\mathrm{d}\vec{S}_{\text{bottom}}$ are perpendicular to $\vec{E}$ their scalar products are

$$\vec{E} \cdot \mathrm{d}\vec{S}_{\text{top}} = \vec{E} \cdot \mathrm{d}\vec{S}_{\text{bottom}} = 0 \ ,$$

because $\mathrm{d}\vec{S}_{\text{shell}}$ is parallel to $\vec{E}$ their scalar product is

$$\vec{E} \cdot \mathrm{d}\vec{S}_{\text{shell}} = E\mathrm{d}S_{\text{shell}} \ ,$$

because the magnitude of electric field E at a distance r is constant, it is valid

$$\int\limits_{S_{\text{shell}}} E\mathrm{d}S_{\text{shell}} = E \int\limits_{S_{\text{shell}}} \mathrm{d}S_{\text{shell}} = E2\pi rl$$

and the electric charge inside the cylinder can be expressed as

$$Q = \lambda l \ .$$

From Gauss' law of electrostatics follows

$$E2\pi rl = \frac{\lambda l}{\epsilon_0} \ ,$$

from which it is possible to express the electric field of the conductor in the place where the charged particle is located

$$E = \frac{\lambda}{2\pi r\epsilon_0} \ .$$

The force acting at this location on a particle with charge Q' will be

$$F = EQ' = \frac{\lambda}{2\pi r\epsilon_0}Q' \ ,$$

after substitution of numerical values

$$F = \frac{3 \cdot 10^{-6}\,\mathrm{C\,m^{-1}}}{2\pi \ . \ 0{,}02\,\mathrm{m} \ . \ 8{,}854 \cdot 10^{-12}\,\mathrm{C^2\,N^{-1}\,m^{-2}}} \ . \ 5 \cdot 10^{-6}\,\mathrm{C} = 13{,}5\,\mathrm{N} \ .$$

4. *Two capacitors with electrical capacitances* $C_1 = 1\,\mu\mathrm{F}$ *and* $C_2 = 2\,\mu\mathrm{F}$ *are connected in series and connected to a voltage source* $U = 600\,\mathrm{V}$. *Calculate the charge and the voltage on each of them. We then disconnect the charged capacitors from the source and each other and reconnect them in parallel by connecting the positive and negative electrodes of the capacitors. Calculate the charge and the voltage will be on each of them after the stabilization.*

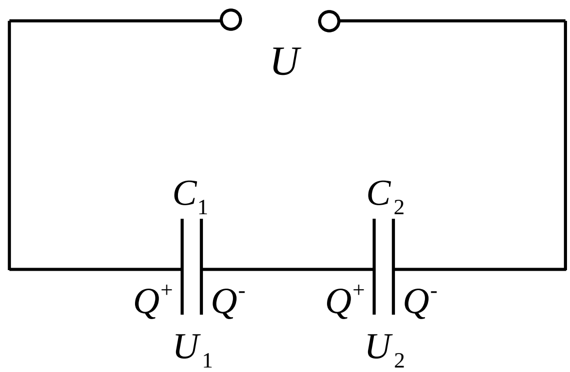

Fig. 4

When capacitors are connected in series (Fig. 27), the charge on the capacitors is equal

$$Q = Q_1 = Q_2$$

and the total voltage on the capacitors is the sum of the voltages on the individual capacitors

$$U = U_1 + U_2\ .$$

From the definition of the capacitance for the voltage follows

$$U = \frac{Q}{C}\ ,$$

using this formula, the sum of the voltages can be written in the form

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2}\ ,$$

from which it is possible to express the resulting capacitance

$$C = \frac{C_1 C_2}{C_1 + C_2}\ ,$$

and then calculates the charge on the capacitors

$$Q = CU = \frac{C_1 C_2}{C_1 + C_2} U \,.$$

After substitution

$$Q = \frac{1\,\mu\mathrm{F}\;.\;2\,\mu\mathrm{F}}{1\,\mu\mathrm{F} + 2\,\mu\mathrm{F}}\;.\;600\,\mathrm{V} = 400\,\mu\mathrm{C}\,,$$

the voltage on the first capacitor will be

$$U_1 = \frac{Q}{C_1}\,,$$

after substitution

$$U_1 = \frac{400\,\mu\mathrm{C}}{1\,\mu\mathrm{F}} = 400\,\mathrm{V}$$

and the voltage on the second capacitor will be

$$U_2 = \frac{Q}{C_2}\,,$$

after substitution

$$U_2 = \frac{400\,\mu\mathrm{C}}{2\,\mu\mathrm{F}} = 200\,\mathrm{V}\,.$$

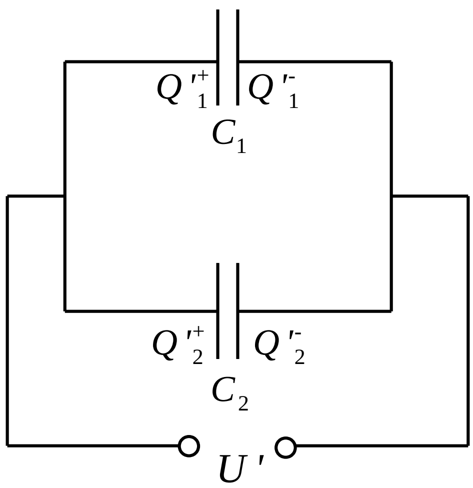

Fig. 5

When the capacitors are connected in parallel (Fig. 28), the voltage on the capacitors is equal

$$U' = U_1' = U_2'$$

and the total charge on the capacitors is the sum of the charges on the individual capacitors

$$Q' = Q'_1 + Q'_2 = 2Q \ .$$

From the definition of the capacitance for the charge follows

$$Q = CU \ ,$$

using this formula, the sum of the charges can be written in the form

$$C'U' = C_1U' + C_2U' \ ,$$

from which it is possible to express the resulting capacitance

$$C' = C_1 + C_2$$

and then calculates the voltage on the capacitors

$$U' = \frac{Q'}{C'} = \frac{2Q}{C_1 + C_2} \ ,$$

after substitution

$$U' = \frac{2 \ . \ 400 \, \mu\mathrm{C}}{1 \, \mu\mathrm{F} + 2 \, \mu\mathrm{F}} = 266{,}7 \, \mathrm{V} \ ,$$

the charge on the first capacitor will be

$$Q'_1 = C_1U' \ ,$$

after substitution

$$Q'_1 = 1 \, \mu\mathrm{F} \ . \ 266{,}7 \, \mathrm{V} = 266{,}7 \, \mu\mathrm{C} \ ,$$

and the charge on the second capacitor will be

$$Q'_2 = C_2U' \ ,$$

after substitution

$$Q'_2 = 2 \, \mu\mathrm{F} \ . \ 266{,}7 \, \mathrm{V} = 533{,}4 \, \mu\mathrm{C} \ .$$

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5. *Calculate the capacitance of the cylindrical capacitor, which is formed by two coaxial conductive cylindrical surfaces in a vacuum. The height of both is* $h = 2 \, \mathrm{cm}$*, the radius of the inner electrode is* $r_1 = 0{,}5 \, \mathrm{mm}$ *and the radius of the outer electrode is* $r_2 = 5 \, \mathrm{mm}$.

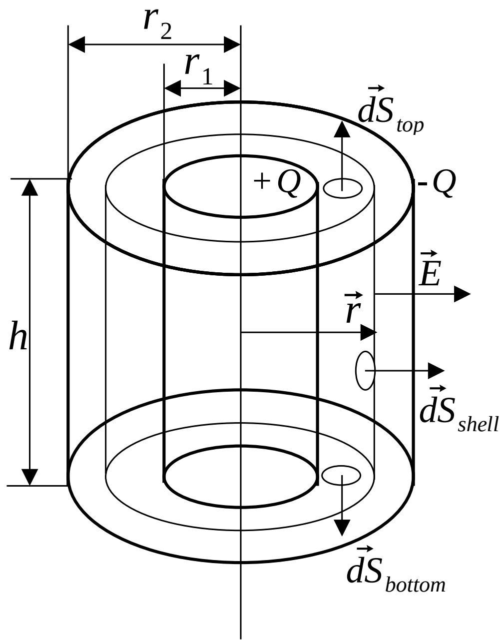

Fig. 6

The electric field between the electrodes of the capacitor can be expressed using the Gauss' law of electrostatics, which states that the electric flux through any closed surface is equal to the ratio of the electric charge inside the closed surface and the electric constant

$$\oint_S \vec{E} \cdot \mathrm{d}\vec{S} = \frac{Q}{\epsilon_0} \; .$$

If the closed surface is chosen as the surface of a cylinder whose axis of symmetry is located on the axis of the capacitor, the total electric flux can be expressed as the sum of the electric fluxes through the upper base, the lower base and the shell of the cylinder

$$\oint_S \vec{E} \cdot \mathrm{d}\vec{S} = \int_{S_{\text{top}}} \vec{E} \cdot \mathrm{d}\vec{S}_{\text{top}} + \int_{S_{\text{bottom}}} \vec{E} \cdot \mathrm{d}\vec{S}_{\text{bottom}} + \int_{S_{\text{shell}}} \vec{E} \cdot \mathrm{d}\vec{S}_{\text{shell}} \; .$$

Because both $\mathrm{d}\vec{S}_{\text{top}}$ and $\mathrm{d}\vec{S}_{\text{bottom}}$ are perpendicular to $\vec{E}$ their scalar products are

$$\vec{E} \cdot \mathrm{d}\vec{S}_{\text{top}} = \vec{E} \cdot \mathrm{d}\vec{S}_{\text{bottom}} = 0 \; ,$$

because $\mathrm{d}\vec{S}_{\text{shell}}$ is parallel to $\vec{E}$ their scalar product is

$$\vec{E} \cdot \mathrm{d}\vec{S}_{\text{shell}} = E \mathrm{d}S_{\text{shell}} \; ,$$

because the magnitude of electric field E at a distance r is constant, it is valid

$$\int_{S_{\text{shell}}} E \mathrm{d}S_{\text{shell}} = E \int_{S_{\text{shell}}} \mathrm{d}S_{\text{shell}} = E 2\pi r h \; .$$

From Gauss' law of electrostatics follows

$$E 2\pi r h = \frac{Q}{\epsilon_0} \ ,$$

from which it is possible to express the electric field as

$$E = \frac{Q}{2\pi r h \epsilon_0} \ .$$

The voltage between the electrodes can be calculated by integrating the electric field

$$U = \int_{\vec{r}_1}^{\vec{r}_2} \vec{E} \cdot \mathrm{d}\vec{r} = \int_{r_1}^{r_2} E \mathrm{d}r \cos 0° = \int_{r_1}^{r_2} \frac{Q}{2\pi r h \epsilon_0} \mathrm{d}r = \frac{Q}{2\pi h \epsilon_0} \int_{r_1}^{r_2} \frac{1}{r} \mathrm{d}r =$$

$$= \frac{Q}{2\pi h \epsilon_0} \left[\ln r\right]_{r_1}^{r_2} = \frac{Q}{2\pi h \epsilon_0} \ln \frac{r_2}{r_1} \ .$$

The electrical capacitance of the capacitor can now be expressed from the definition

$$C = \frac{Q}{U} = \frac{2\pi h \epsilon_0}{\ln \frac{r_2}{r_1}} \ ,$$

after substitution

$$C = \frac{2\pi \ . \ 0{,}02\,\mathrm{m} \ . \ 8{,}854 \cdot 10^{-12}\,\mathrm{C^2\,N^{-1}\,m^{-2}}}{\ln \frac{5 \cdot 10^{-3}\,\mathrm{m}}{0{,}5 \cdot 10^{-3}\,\mathrm{m}}} = 4{,}83 \cdot 10^{-13}\,\mathrm{F} = 0{,}483\,\mathrm{pF} \ .$$

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6. *Calculate the capacitance of the spherical capacitor, which consists of two concentric conductive spherical surfaces with radius* $r_1 = 3\,\mathrm{cm}$ *and* $r_2 = 4\,\mathrm{cm}$*, if the medium between them is filled with a dielectric with relative permittivity* $\epsilon_r = 2{,}6$*. What will be the charge on the electrodes if the capacitor is connected to voltages* $U = 600\,\mathrm{V}$ *and what will be the energy of the capacitor?*

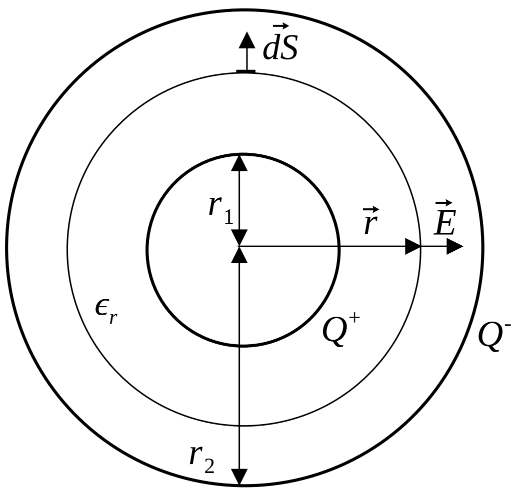

Fig. 7

The electric field between the electrodes of the capacitor can be expressed using the Gauss' law of electrostatics

$$\oint_S \vec{E} \cdot \mathrm{d}\vec{S} = \frac{Q}{\epsilon_0 \epsilon_r} \, .$$

If the closed surface is chosen so that it is the surface of a sphere with the center located in the center of the capacitor, the electric field vector will have the same direction as the surface element vector, and the magnitude of the electric field will be constant everywhere on this surface. Therefore, it will apply

$$\oint_S \vec{E} \cdot \mathrm{d}\vec{S} = \oint_S E \mathrm{d}S \cos 0^\circ = E \int_S \mathrm{d}S = ES = E 4\pi r^2 \, .$$

Then the Gauss' law of electrostatics will imply

$$E 4\pi r^2 = \frac{Q}{\epsilon_0 \epsilon_r} \, ,$$

from which it is possible to express the electric field

$$E = \frac{Q}{4\pi r^2 \epsilon_0 \epsilon_2} \, ,$$

and the voltage between the electrodes can be calculated by integrating the electric field

$$U = \int_{\vec{r}_1}^{\vec{r}_2} \vec{E} \cdot \mathrm{d}\vec{r} = \int_{r_1}^{r_2} E \mathrm{d}r \cos 0^\circ = \int_{r_1}^{r_2} \frac{Q}{4\pi r^2 \epsilon_0 \epsilon_r} \mathrm{d}r = \frac{Q}{4\pi \epsilon_0 \epsilon_r} \int_{r_1}^{r_2} \frac{1}{r^2} \mathrm{d}r =$$

$$= \frac{Q}{4\pi \epsilon_0 \epsilon_r} \left[-\frac{1}{r} \right]_{r_1}^{r_2} = \frac{Q}{4\pi \epsilon_0 \epsilon_r} \left(\frac{1}{r_1} - \frac{1}{r_2} \right) = \frac{Q}{4\pi \epsilon_0 \epsilon_r} \frac{r_2 - r_1}{r_1 r_2} \, .$$

The electrical capacitance of the capacitor can now be expressed from the definition

$$C = \frac{Q}{U} = 4\pi \epsilon_0 \epsilon_r \frac{r_1 r_2}{r_2 - r_1} \, ,$$

after substitution

$$C = 4\pi \, . \, 8{,}854 \cdot 10^{-12} \, \mathrm{C}^2 \, \mathrm{N}^{-1} \, \mathrm{m}^{-2} \, . \, 2{,}6 \, . \, \frac{0{,}03 \, \mathrm{m} \, . \, 0{,}04 \, \mathrm{m}}{0{,}04 \, \mathrm{m} - 0{,}03 \, \mathrm{m}} = 3{,}47 \cdot 10^{-11} \, \mathrm{F} = 34{,}7 \, \mathrm{pF} \, .$$

If the capacitor is connected to an electric voltage $U = 300 \, \mathrm{V}$, the charge on its electrodes will be

$$Q = CU = 3{,}47 \cdot 10^{-11} \, \mathrm{F} \, . \, 600 \, \mathrm{V} = 2{,}08 \cdot 10^{-8} \, \mathrm{C} = 20{,}8 \, \mathrm{nC}$$

and the energy of the electric field of the capacitor will be

$$W = \frac{1}{2} C U^2 = \frac{1}{2} \, . \, 3{,}47 \cdot 10^{-11} \, \mathrm{F} \, . \, (600 \, \mathrm{V})^2 = 6{,}25 \cdot 10^{-6} \, \mathrm{J} = 6{,}25 \, \mu\mathrm{J} \, .$$

7. *Calculate the electric field in an aluminum conductor in the shape of a straight cylinder with a radius* $r_0 = 2{,}5\,\mathrm{mm}$ *and a length* $L = 1\,\mathrm{m}$*, when a stationary electric current of* $I = 10\,\mathrm{A}$ *flows through it. What will be the voltage at the ends of this conductor and what electric charge will flow through the conductor during time* $t' = 10\,\mathrm{s}$*? The resistivity of aluminum is* $\rho = 2{,}828 \cdot 10^{-8}\,\Omega\,\mathrm{m}$.

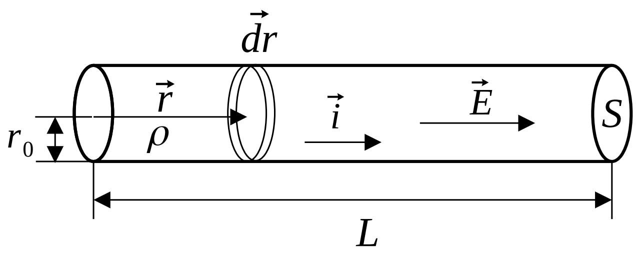

Fig. 8

The solution can be found using Ohm's law in differential form

$$\vec{i} = \gamma \vec{E} ,$$

where γ is the conductivity of the conductor

$$\gamma = \frac{1}{\rho} ,$$

and $\vec{i}$ is the electric current density, its magnitude is

$$i = \frac{I}{S} = \frac{I}{\pi r_0^2} .$$

In the scalar form it is possible to write

$$i = \gamma E ,$$

from which it is possible to express the magnitude of the electric field

$$E = \frac{i}{\gamma} = \frac{1}{\gamma}\frac{I}{\pi r_0^2} = \rho \frac{I}{\pi r_0^2} ,$$

after substitution

$$E = 2{,}828 \cdot 10^{-8}\,\Omega\,\mathrm{m} \,.\, \frac{10\,\mathrm{A}}{\pi \,.\, (0{,}0025\,\mathrm{m})^2} = 1{,}44 \cdot 10^{-2}\,\mathrm{V\,m^{-1}} = 14{,}4\,\mathrm{mV\,m^{-1}} .$$

The voltage between the ends of the conductor will be

$$U = \int_{\vec{r}_1}^{\vec{r}_2} \vec{E} \cdot \mathrm{d}\vec{r} = \int_0^L E\mathrm{d}r = EL = 1{,}44 \cdot 10^{-2}\,\mathrm{V\,m^{-1}} \,.\, 1\,\mathrm{m} = 1{,}44 \cdot 10^{-2}\,\mathrm{V} = 14{,}4\,\mathrm{mV}$$

and the electric charge that will flow in time t^* will be

$$Q^* = \int_0^{t^*} I\mathrm{d}t = It^* = 10\,\mathrm{A} \,.\, 10\,\mathrm{s} = 100\,\mathrm{C} \,.$$

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8. *The parallel plate capacitor has electrodes with an area $S = 16\,\mathrm{cm}^2$, the distance between the electrodes $d = 0{,}2\,\mathrm{cm}$ and the space between the electrodes is filled with a dielectric with a relative permittivity $\epsilon_r = 6{,}9$. Calculate the capacitance of the capacitor, the charge on the electrodes, the electric field, the electric displacement field, the energy density and the energy of the electric field if the capacitor is connected to an electric voltage $U = 300\,\mathrm{V}$.*

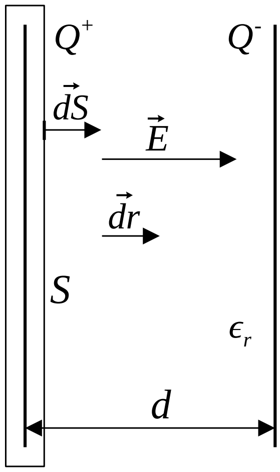

Fig. 9

he electric field between the electrodes of the capacitor can be expressed using the Gauss' law of electrostatics

$$\oint_S \vec{E} \cdot \mathrm{d}\vec{S} = \frac{Q}{\epsilon_r \epsilon_0} \,,$$

the closed area around the electrode has a total area of $2S$, but the electric field is only on one side of it. The electric field is constant and has the direction of the surface element, therefore

$$\oint_S \vec{E} \cdot \mathrm{d}\vec{S} = \oint_S E\mathrm{d}S = E \oint_S \mathrm{d}S = ES \,,$$

which gives the equation

$$ES = \frac{Q}{\epsilon_r \epsilon_0} \ ,$$

from which it is possible to express the electric field

$$E = \frac{Q}{S \epsilon_r \epsilon_0}$$

and the voltage between the electrodes will be

$$U = \int_{\vec{r}_1}^{\vec{r}_2} \vec{E} \cdot \mathrm{d}\vec{r} = \int_{r_1}^{r_2} E \mathrm{d}r = E \int_{r_1}^{r_2} \mathrm{d}r = Ed = \frac{Q}{S \epsilon_r \epsilon_0} d \ .$$

The capacitance of the parallel plate capacitor will be

$$C = \frac{Q}{U} = \frac{S \epsilon_r \epsilon_0}{d} = \frac{16 \cdot 10^{-4}\,\mathrm{m}^2 \ . \ 6{,}9 \ . \ 8{,}854 \cdot 10^{-12}\,\mathrm{C}^2\,\mathrm{N}^{-1}\,\mathrm{m}^{-2}}{0{,}002\,\mathrm{m}} =$$

$$= 4{,}89 \cdot 10^{-11}\,\mathrm{F} = 48{,}9\,\mathrm{pF} \ .$$

The charge on the electrodes will be

$$Q = CU = 4{,}89 \cdot 10^{-11}\,\mathrm{F} \ . \ 300\,\mathrm{V} = 1{,}47 \cdot 10^{-8}\,\mathrm{C} = 14{,}7\,\mathrm{nC} \ .$$

The electric field will be

$$E = \frac{U}{d} = \frac{300\,\mathrm{V}}{0{,}002\,\mathrm{m}} = 1{,}5 \cdot 10^5\,\mathrm{V\,m}^{-1} = 150\,\mathrm{kV\,m}^{-1} \ .$$

The electric displacement field will be

$$D = \epsilon_r \epsilon_0 E = 6{,}9 \ . \ 8{,}854 \cdot 10^{-12}\,\mathrm{C}^2\,\mathrm{N}^{-1}\,\mathrm{m}^{-2} \ . \ 1{,}5 \cdot 10^5\,\mathrm{V\,m}^{-1} =$$

$$= 9{,}16 \cdot 10^{-6}\,\mathrm{C\,m}^{-2} = 9{,}16\,\mu\mathrm{C\,m}^{-2} \ .$$

The energy density of the electric field will be

$$w = \frac{1}{2} ED = \frac{1}{2} \ . \ 1{,}5 \cdot 10^5\,\mathrm{V\,m}^{-1} \ . \ 9{,}16 \cdot 10^{-6}\,\mathrm{C\,m}^{-2} = 0{,}687\,\mathrm{J\,m}^{-3}$$

The energy of the electric field will be

$$W = \frac{1}{2} CU^2 = \frac{1}{2} \ . \ 4{,}89 \cdot 10^{-11}\,\mathrm{F} \ . \ (300\,\mathrm{V})^2 = 2{,}2 \cdot 10^{-6}\,\mathrm{J} = 2{,}2\,\mu\mathrm{J} \ .$$

9. *Two resistors with resistances* $R_1 = 4\,\Omega$ *and* $R_2 = 12\,\Omega$ *are connected in parallel and connected to a source with electromotive voltage* $U_e = 9\,\mathrm{V}$ *and internal resistance* $R_i = 1{,}5\,\Omega$. *What electric currents are in the individual branches of the circuit?*

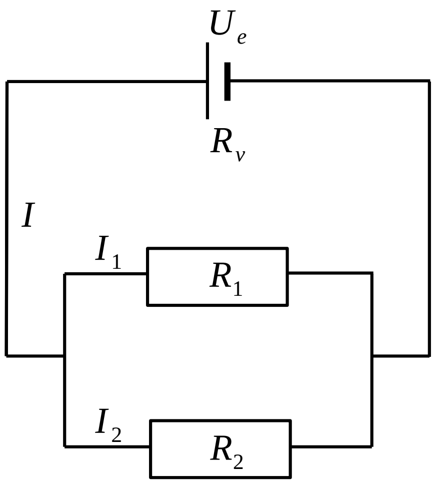

Fig. 10

For resistors connected in parallel (Fig. 33) with resistances R_1 and R_2, the following applies

$$U = U_1 = U_2 \ ,$$

$$I = I_1 + I_2 \ .$$

From Ohm's law

$$I = \frac{U}{R} \ ,$$

it follows

$$\frac{U}{R} = \frac{U}{R_1} + \frac{U}{R_2}$$

and the resulting resistance will be

$$R = \frac{R_1 R_2}{R_1 + R_2} \ .$$

The following applies to the electromotive voltage of the source

$$U_e = (R + R_v) I \ ,$$

which implies for electric current

$$I = \frac{U_e}{R + R_v} = \frac{U_e}{\frac{R_1 R_2}{R_1 + R_2} + R_v} = \frac{U_e (R_1 + R_2)}{R_1 R_2 + R_v (R_1 + R_2)} \ ,$$

after substitution

$$I = \frac{9\,\mathrm{V} \ . \ (4\,\Omega + 12\,\Omega)}{4\,\Omega \ . \ 12\,\Omega + 1{,}5\,\Omega \ . \ (4\,\Omega + 12\,\Omega)} = 2\,\mathrm{A} \ .$$

The electric current through the resistor with resistance R_1 will be

$$I_1 = \frac{U}{R_1} = \frac{RI}{R_1} = \frac{\frac{R_1 R_2}{R_1 + R_2} I}{R_1} = \frac{R_2 I}{R_1 + R_2} \ ,$$

after substitution

$$I_1 = \frac{12\,\Omega \ . \ 2\,\mathrm{A}}{4\,\Omega + 12\,\Omega} = 1{,}5\,\mathrm{A} \ .$$

The electric current through the resistor with resistance R_2 will be

$$I_2 = \frac{U}{R_2} = \frac{RI}{R_2} = \frac{\frac{R_1 R_2}{R_1 + R_2} I}{R_2} = \frac{R_1 I}{R_1 + R_2} \ ,$$

after substitution

$$I_2 = \frac{4\,\Omega \ . \ 2\,\mathrm{A}}{4\,\Omega + 12\,\Omega} = 0{,}5\,\mathrm{A} \ .$$

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10. *The electric current in the conductor, whose electrical resistance is $R = 10\,\Omega$, decrease linearly from the value $I_0 = 2\,\mathrm{A}$ to the zero value during the time $t_0 = 3\,\mathrm{s}$. What heat was generated in the conductor during this time and what electric charge flowed through the conductor during this time?*

The electric current decreased linearly from the value I_0, that is

$$I = I_0 - kt \ ,$$

to a zero value at time t_0, so the following applies

$$0 = I_0 - kt_0 \ ,$$

from which it is possible to express the constant

$$k = \frac{I_0}{t_0} \ ,$$

the electric current will therefore vary with time as

$$I = I_0 - \frac{I_0}{t_0} t \,.$$

The following applies to heat

$$\mathrm{d}Q = P \mathrm{d}t \,,$$

where the power of the electric current will be

$$P = UI = RI^2 \,.$$

The heat generated in the conductor during time t_0 will be

$$Q = \int_0^{t_0} P \mathrm{d}t = \int_0^{t_0} RI^2 \mathrm{d}t = R \int_0^{t_0} \left(I_0 - \frac{I_0}{t_0} t \right)^2 \mathrm{d}t = R \int_0^{t_0} \left(I_0^2 - 2\frac{I_0^2}{t_0} t + \frac{I_0^2}{t_0^2} t^2 \right) \mathrm{d}t =$$

$$= R \left[I_0^2 t - \frac{I_0^2}{t_0} t^2 + \frac{I_0^2}{t_0^2} \frac{t^3}{3} \right]_0^{t_0} = \frac{R I_0^2 t_0}{3} \,,$$

after substitution

$$Q = \frac{10\,\Omega \,.\, (2\,\mathrm{A})^2 \,.\, 3\,\mathrm{s}}{3} = 40\,\mathrm{J} \,.$$

The following applies to electric charge

$$\mathrm{d}Q = I \mathrm{d}t \,.$$

The electric charge that flows through the conductor in time t_0, will be

$$q = \int_0^{t_0} I \mathrm{d}t = \int_0^{t_0} \left(I_0 - \frac{I_0}{t_0} t \right) \mathrm{d}t = \left[I_0 t - \frac{I_0}{t_0} \frac{t^2}{2} \right]_0^{t_0} = \frac{I_0 t_0}{2} \,,$$

after substitution

$$q = \frac{2\,\mathrm{A} \,.\, 3\,\mathrm{s}}{2} = 3\,\mathrm{C} \,.$$